Velocity Motion Model (cont)

Velocity Motion Model

avg of $\begin{bmatrix} x' \\ y' \end{bmatrix}$ & $\begin{bmatrix} x \\ y \end{bmatrix}$

- center of circle

unknown distance

$$\begin{pmatrix} x^* \\ y^* \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -\lambda \sin\theta \\ \lambda \cos\theta \end{pmatrix} = \begin{pmatrix} \frac{x+x'}{2} + \mu(y-y') \\ \frac{y+y'}{2} + \mu(x'-x) \end{pmatrix}$$

vector pointing towards the ICC

where

$$\mu = \frac{1}{2} \frac{(x-x')\cos\theta + (y-y')\sin\theta}{(y-y')\cos\theta - (x-x')\sin\theta}$$

2 sets of equations

2 unknowns (μ, λ)

solve for μ

μ unknown scalar

Velocity Motion Model

commanded velocities

Algorithm motion_model_velocity(x_t, u_t, x_{t-1}):

$$\mu = \frac{1}{2}\,\frac{(x-x')\cos\theta + (y-y')\sin\theta}{(y-y')\cos\theta - (x-x')\sin\theta}$$

$$x^* = \frac{x+x'}{2} + \mu(y-y')$$

compute ICC by intersecting 2 lines

$$y^* = \frac{y+y'}{2} + \mu(x'-x)$$

$$r^* = \sqrt{(x-x^*)^2 + (y-y^*)^2}$$

distance to ICC

$$\Delta\theta = \operatorname{atan2}(y'-y^*, x'-x^*) - \operatorname{atan2}(y-y^*, x-x^*)$$

change in heading

$$\hat{v} = \frac{\Delta\theta}{\Delta t}\, r^*$$

$$\hat{\omega} = \frac{\Delta\theta}{\Delta t}$$

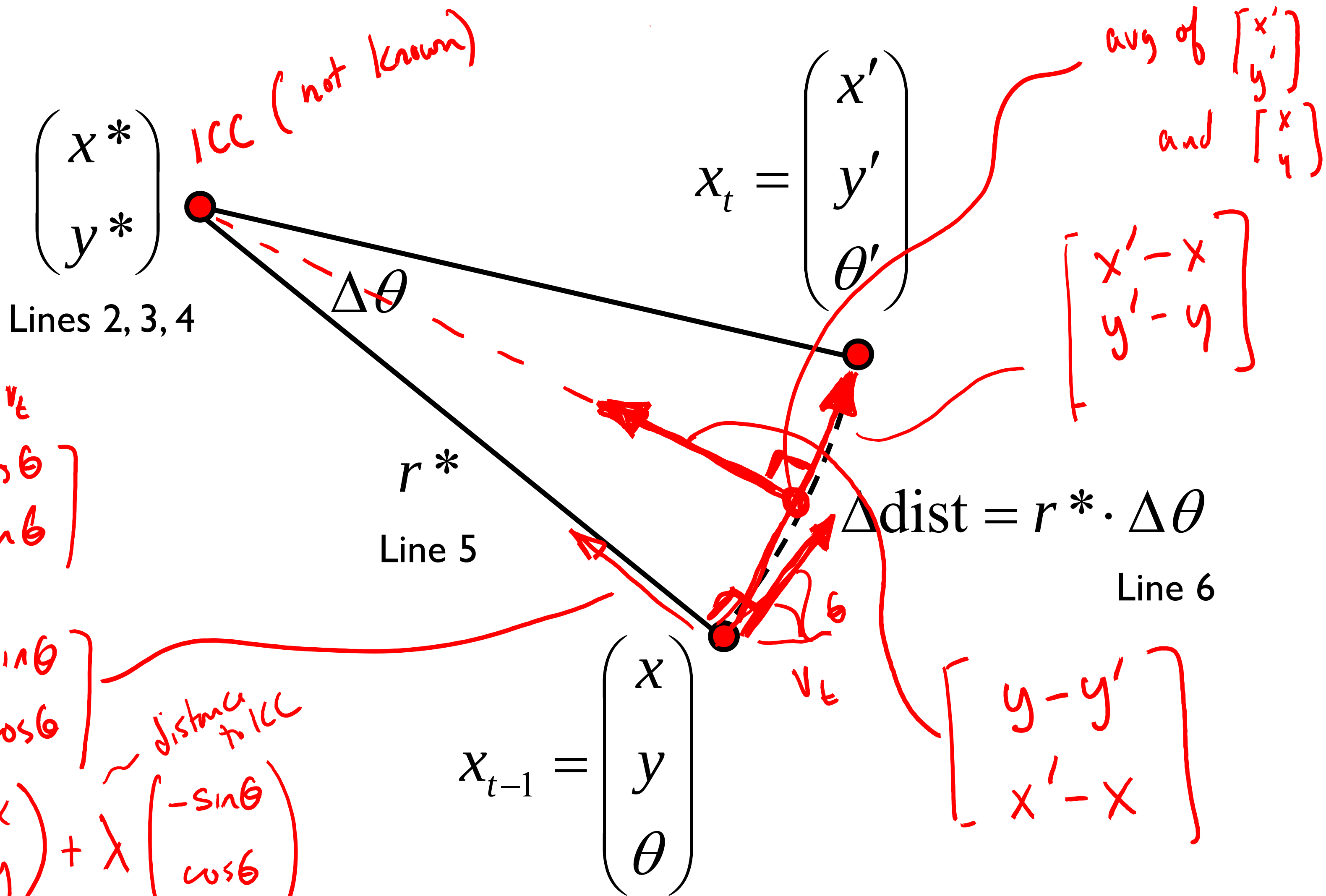
$$\hat{\gamma} = \frac{\theta'-\theta}{\Delta t} - \hat{\omega}$$

probability densities

return $\mathbf{prob}(v-\hat{v}, \alpha_1 v^2 + \alpha_2 \omega^2) \cdot \mathbf{prob}(\omega-\hat{\omega}, \alpha_3 v^2 + \alpha_4 \omega^2) \cdot \mathbf{prob}(\hat{\gamma}, \alpha_5 v^2 + \alpha_6 \omega^2)$

Velocity Motion Model

- rotation of Δθ about (x*, y*) from (x, y) to (x’, y’) in time Δt



unit vector in dir v_t

$\hat{V}_t = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$

$\begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$

~ distance to ICC

$\begin{pmatrix} x^* \\ y^* \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \lambda \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}$

Velocity Motion Model

- given $\Delta\theta$ and Δdist we can compute the velocities needed to generate the motion

$r^* \Delta\theta$

$$\hat{u}_t = \begin{pmatrix} \hat{v}_t \\ \hat{\omega}_t \end{pmatrix} = \begin{pmatrix} \Delta \text{dist} \,/\, \Delta t \\ \Delta\theta \,/\, \Delta t \end{pmatrix}$$

Steps 7, 8

given the computed ICC

- notice what the algorithm has done
  - it has used an inverse motion model to compute the control vector that would be needed to produce the motion from x_{t-1} to x_t
  - in general, the computed control vector will be different from the actual control vector u_t

Velocity Motion Model

- recall that we want the posterior conditional density

$$p(x_t \mid u_t, x_{t-1})$$

  of the control action u_t carrying the robot from pose $x_{t\text{-}1}$ to x_t in time Δt
- so far the algorithm has computed the required control action $\hat{u}_t$ needed to carry the robot from position (x y) to position (x' y')
  - the control action has been computed assuming the robot moves on a circular arc

Velocity Motion Model

- the computed heading of the robot is $$\hat{\theta} = \underbrace{\theta}_{x_{t-1}} + \underbrace{\Delta\theta}_{\text{from Step 6}}$$
  assuming robot moved with control vector $\hat{u} = \begin{bmatrix} \hat{v} \\ \hat{\omega} \end{bmatrix}$
- the heading should be $$\theta' \quad \text{from } x_t$$
- the difference is $$\theta_{\text{err}} = \theta' - \hat{\theta}$$ $$= \theta' - \theta - \Delta\theta$$
- or expressed as an angular velocity

$$\gamma_{\text{err}} = \frac{\theta_{\text{err}}}{\Delta t}$$

$$= \frac{\theta' - \theta}{\Delta t} - \hat{\omega}$$

Line 9,
Eq 5.25, 5.28

Velocity Motion Model

- similarly, we can compute the errors of the computed linear and rotational velocities

u_t Line 7

$$v_{\text{err}} = v - \hat{v}$$

$$= v - \frac{\Delta \text{dist}}{\Delta t}$$

u_t Line 8

$$\omega_{\text{err}} = \omega - \hat{\omega}$$

$$= \omega - \frac{\Delta \theta}{\Delta t}$$

Velocity Motion Model

- if we assume that the robot has independent control over its controlled linear and angular velocities then the joint density of the errors is

probably not a great assumption

$$\underbrace{p(v_{\text{err}}, \omega_{\text{err}}, \gamma_{\text{err}})} = p(v_{\text{err}})\ p(\omega_{\text{err}})\ p(\gamma_{\text{err}})$$

probability that we observe v_{err} & ω_{err} & γ_{err}

- what do the individual densities look like?

Velocity Motion Model

$\mu = 0$

- the most common noise model is additive zero-mean noise, i.e.

$$\begin{pmatrix} \hat{v} \\ \hat{\omega} \end{pmatrix} = \begin{pmatrix} v \\ \omega \end{pmatrix} + \begin{pmatrix} v_{\text{noise}} \\ \omega_{\text{noise}} \end{pmatrix}$$

actual velocity	commanded velocity	noise

random variables

- we need to decide on other characteristics of the noises
  - "spread" variance
  - "skew" skew
  - "peakedness" kurtosis
- typically, only the variance is specified
  - the true variance is typically unknown

Velocity Motion Model

- the textbook assumes that the variances can be modeled as

$$\mathrm{var}(v_{\mathrm{noise}}) = \alpha_1 v^2 + \alpha_2 \omega^2$$

$$\mathrm{var}(\omega_{\mathrm{noise}}) = \alpha_3 v^2 + \alpha_4 \omega^2$$

more noise as robot moves faster

Eq 5.10

where the α_i are robot specific error parameters

- the less accurate the robot the larger the α_i

Velocity Motion Model

- a robot travelling on a circular arc has no independent control over its heading
  - the heading must be tangent to the arc

$$\theta' = \theta + \hat{\omega}\,\Delta t$$

$p(\theta') = 0$ because the robot heading must be $\hat{\theta}$ if it moved with smooth rolling motion

- this is problematic if you have a noisy commanded angular velocity ω
- thus, we assume that the final heading is actually given by

$$\theta' = \theta + \hat{\omega}\,\Delta t + \hat{\gamma}\,\Delta t \qquad \text{Eq 5.14}$$

random in-place rotation at x_t

where $\hat{\gamma}$ is the angular velocity of the robot spinning in place

Velocity Motion Model

- the book assumes that

i.e., robot heading is on average $\hat{\theta}$

$$\hat{\gamma} = 0 + \gamma_{\text{noise}}$$

actual velocity

noise

where

$$\text{var}(\gamma_{\text{noise}}) = \alpha_5 v^2 + \alpha_6 \omega^2 \qquad \text{Eq 5.15}$$

we now have assumed distribution of

$\hat{v}, \hat{\omega}, \hat{\gamma}$